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### SUMMARY

An efficient technique of microcomputer modeling is presented for designing linear and nonlinear circuits. The computer model is valid for pulse forming networks (PFN's) which feed both linear and nonlinear loads. These PFN's may resemble classical voltage or current fed one terminal-pair Guillemin networks, but are in fact, the general case of loaded two terminal-pair networks including most nonuniform lumped transmission lines. Before building systems using PFN's it is desirable to calculate their response, however, closed form analytic solutions are impractical and other computer approaches such as SCEPTRE and SPICE require a computer with a large capacity. An algorithm is presented that can be applied to pulsed discharge circuits employing PFN's and solved on a microcomputer such as an Apple or HP-85. Results are presented for a circuit consisting of an initially charged capacitor with stray inductance in series with a PFN discharging into either a linear or a nonlinear load.

### INTRODUCTION

Many programs have been developed for network analysis of pulsed circuits such as SCEPTRE that require a main-frame computer to handle the program and are expensive to run because of the time needed to obtain a solution. The technique described in this paper has the advantage of being efficient enough to be programmed on a microcomputer. The microcomputer is more readily available and much less expensive to operate and produces a good approximation to the desired waveform in a short solution time.

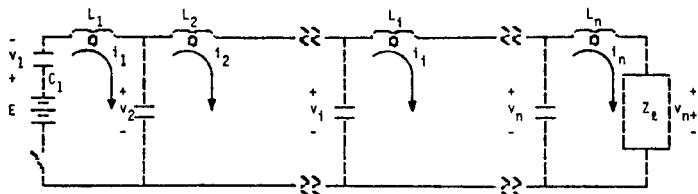


FIGURE 1. n Section LC Network

### ALGORITHM

Given the n-section LC network in Figure 1, a first order differential equation is found for each current by summing the voltage drops around each loop and solving for the derivative of the current. The differential equation for the first loop is given in Equation 1,

$$\frac{di_1}{dt} \Big|_{t=t_{k-1}} = \frac{E - v_1 \Big|_{t=t_{k-1}} - v_2 \Big|_{t=t_{k-1}}}{L_1} \quad (1)$$

where  $k = 1, 2, 3, \dots, n$

$$\text{and } t_k = t_{k-1} + \Delta t$$

$$\Delta t = \frac{t_k - t_0}{K}$$

where  $t_0$  = initial time, and

$t_K$  = last time the network is to be solved

The differential equation for the remaining loops is shown in Equation 2.

$$\frac{di_i}{dt} \Big|_{t=t_{k-1}} = \frac{v_i \Big|_{t=t_{k-1}} - v_{i+1} \Big|_{t=t_{k-1}}}{L_i} \quad (2)$$

for  $i = 2, 3, 4, \dots, n$

With these equations and knowing all the initial values of current and voltage at time  $t_0$  a numerical approximation can be made for each current at the next time interval. Eulers method is employed which uses the first two terms of a Taylor series expansion. The general equation for this approximation is shown in Equation 3.

$$i_i \Big|_{t=t_k} = i_i \Big|_{t=t_{k-1}} + \frac{di_i}{dt} \Big|_{t=t_{k-1}} \times \Delta t \quad (3)$$

for  $i = 1, 2, 3, \dots, n$

With the new values of current, the capacitor voltages for the next point in time can be found by numerical integration. The general equation is shown in Equation 4.

$$v_i \Big|_{t=t_k} = v_i \Big|_{t=t_{k-1}} + \frac{i_i \Big|_{t=t_k} - i_{i-1} \Big|_{t=t_k}}{C_i} \times \Delta t \quad (4)$$

for  $i = 1, 2, 3, \dots, n$

This method of integration differs from the rectangular rule in that it approximates the area with the value of the function at time  $t_{k-1}$  multiplied by the increment size.

It is assumed that the voltage across the load,  $v_{n+1}$ , is defined as a function of any known value or parameter in the network at time  $t_k$  or before, for instance;

$$\begin{aligned} v_{n+1} \Big|_{t=t_k} &= Z_L \times i_n \Big|_{t=t_k} \\ v_{n+1} \Big|_{t=t_k} &= Z_L \Big|_{t=t_k} \times i_n \Big|_{t=t_k} \\ v_{n+1} \Big|_{t=t_k} &\equiv f(i_n \Big|_{t=t_k}) \end{aligned}$$

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These are some possible forms of defining the load voltage.

With the case of a purely resistive load the solution is,

$$v_{n+1} \Big|_{t=t_k} = R_L \times i_n \Big|_{t=t_k}$$

Once all the currents and voltages are found for time,  $t_1$ , the process is repeated for time,  $t_2$  and so on.

In most cases, defining loop currents (as opposed to branch currents) and following the paths as was done for the network in this section, gives the least complicated and shortest number of equations.

#### CALCULATED RESPONSE

Taking the network in Figure 2 as an example, the differential equations for current  $i_1$ ,  $i_2$ , and  $i_3$  acquired by summing the voltage drops around the loops are shown in Equations 5, 6, and 7, respectively.

$$\frac{di_1}{dt} \Big|_{t=t_{k-1}} = \left[ \frac{E - v_1 \Big|_{t=t_{k-1}} - v_2 \Big|_{t=t_{k-1}} - v_3 \Big|_{t=t_{k-1}}}{L_1} - v_4 \Big|_{t=t_{k-1}} \right] / [L_1] \quad (5)$$

$$\frac{di_2}{dt} \Big|_{t=t_{k-1}} = \frac{v_2 \Big|_{t=t_{k-1}}}{L_2} \quad (6)$$

$$\frac{di_3}{dt} \Big|_{t=t_{k-1}} = \frac{v_3 \Big|_{t=t_{k-1}}}{L_3} \quad (7)$$

The approximation of each current is given in Equations 8, 9 and 10.

$$i_1 \Big|_{t=t_k} = i_1 \Big|_{t=t_{k-1}} + \frac{di_1}{dt} \Big|_{t=t_{k-1}} \times \Delta t \quad (8)$$

$$i_2 \Big|_{t=t_k} = i_2 \Big|_{t=t_{k-1}} + \frac{di_2}{dt} \Big|_{t=t_{k-1}} \times \Delta t \quad (9)$$

$$i_3 \Big|_{t=t_k} = i_3 \Big|_{t=t_{k-1}} + \frac{di_3}{dt} \Big|_{t=t_{k-1}} \times \Delta t \quad (10)$$

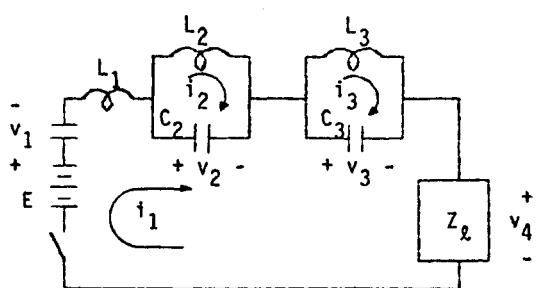


FIGURE 2. Three Section LC Network

The approximation of  $v_1$ ,  $v_2$ , and  $v_3$  is given in Equations 11, 12, and 13, respectively.

$$v_1 \Big|_{t=t_k} = v_1 \Big|_{t=t_{k-1}} + \frac{i_1 \Big|_{t=t_k}}{C_1} \times \Delta t \quad (11)$$

$$v_2 \Big|_{t=t_k} = v_2 \Big|_{t=t_{k-1}} + \frac{i_2 \Big|_{t=t_k}}{C_2} \times \Delta t \quad (12)$$

$$v_3 \Big|_{t=t_k} = v_3 \Big|_{t=t_{k-1}} + \frac{i_3 \Big|_{t=t_k}}{C_3} \times \Delta t \quad (13)$$

For the linear case of  $Z_L = R_L$ ,<sup>5</sup> (a classical voltage fed Guillemin type A network), the voltage across the load,  $v_4$  is given in Equation 14.

$$v_4 \Big|_{t=t_k} = R_L \times i_1 \Big|_{t=t_k} \quad (14)$$

This algorithm was programmed on an HP-85 for parameter values designed to create a 10  $\mu$ s pulse with a 10 ohm resistive load and a charging voltage,  $E$  of 50 volts. Other than the charging voltage, all values of current and voltage at time  $t_0$  are zero and the time increment,  $\Delta t$  is 0.1  $\mu$ s.

To verify the technique an experimental model was constructed and its load voltage was recorded. Figure 3 shows the computer and experimental voltage waveforms across the linear load. Error due to the computer technique is of interest, however, it was difficult to distinguish between this error and errors due to stray inductance and capacitive coupling.

The accuracy of this technique depends on the time increment, however with an increment of only 1/100th of the pulse width the solution takes approximately 44 seconds and the percent error on the average of the amplitudes of the first five peaks and dips of the waveform in Figure 3 was less than 6%.

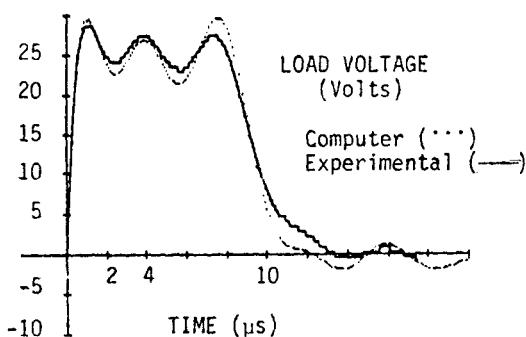


FIGURE 3. Computer & Experimental Voltage Across the Linear Load (10  $\Omega$ )

In order to further demonstrate the capabilities of this technique, the computer waveform shown in Figure 3 is compared to the waveform obtained by analyzing the same network using the SCEPTRE program. This comparison is shown in Figure 4.

The root mean square error of the deviation between SCEPTRE results and results obtained by the technique described here is  $\pm 1.07$  Volts.

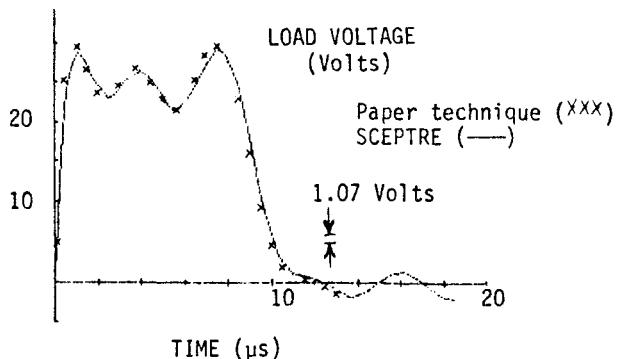


FIGURE 4. Comparison Of The Technique To SCEPTRE

The output waveforms of this network with a nonlinear load consisting of a 100 ohm resistor shunted by 10 series-connected 1N914 diodes were experimentally recorded. The voltage waveforms are shown in Figure 5 and the current waveforms are shown in Figure 6.

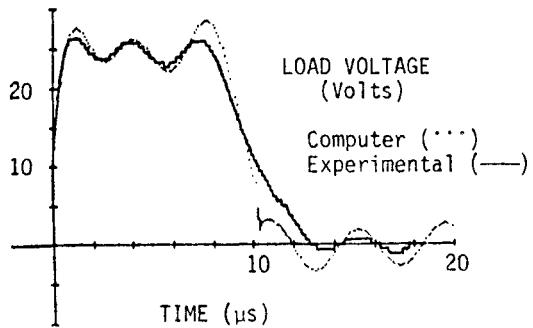


FIGURE 5. Computer & Experimental Voltage Across The Nonlinear Load

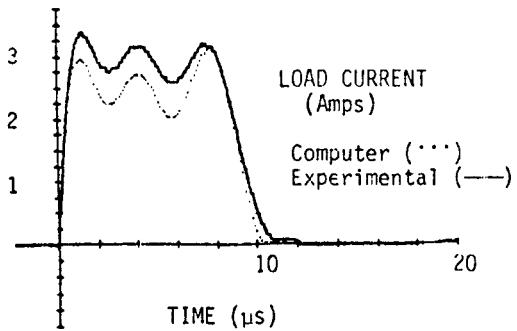


FIGURE 6. Computer & Experimental Current Through The Nonlinear Load

Discrepancies between computer and experimental waveforms can be attributed to the model of the nonlinear load itself, which was developed only from dc characteristics of the load, along with the errors mentioned in the linear case. On the average, the percent error of the amplitudes of the first five peaks and dips of the waveform in Figure 5 is less than 4% and the solution time is approximately 51 seconds. Figures 7, 8 and 9 show the output voltage and their solution time with  $\Delta t = 0.01$   $\mu$ s,  $0.15$   $\mu$ s and  $0.2$   $\mu$ s respectively. It should be noted that with a time

increment of  $0.01$   $\mu$ s the percent error is less than 4% and is only slightly lower than that of  $\Delta t = 0.1$   $\mu$ s, yet the solution time is 5 minutes and 45 seconds longer. If  $\Delta t$  becomes too large so that in that time period the voltage changes a great amount the computer waveforms will oscillate. When this occurs the solution time actually is longer than the runs with smaller time increments because it takes a longer time to plot the oscillations. This occurs when  $\Delta t$  is greater than  $0.8$   $\mu$ s in the linear case and greater than  $0.15$   $\mu$ s in the nonlinear case. The reason this value of  $\Delta t$  is so much smaller in the nonlinear case presented here, is because at low currents the nonlinear load acts as a 100 ohm resistor and consequently the voltage changes ten times more for each value of current than in the linear  $10 \Omega$  case.

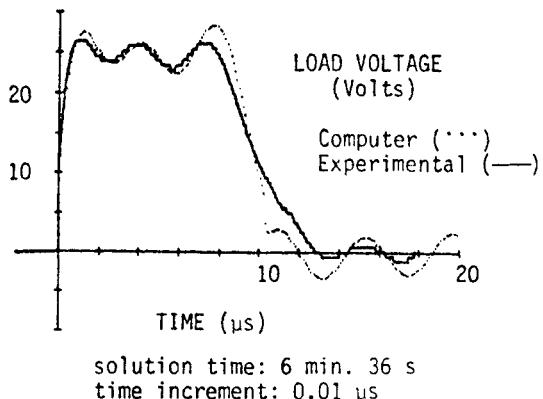


FIGURE 7. Computer & Experimental Voltage Across The Nonlinear Load

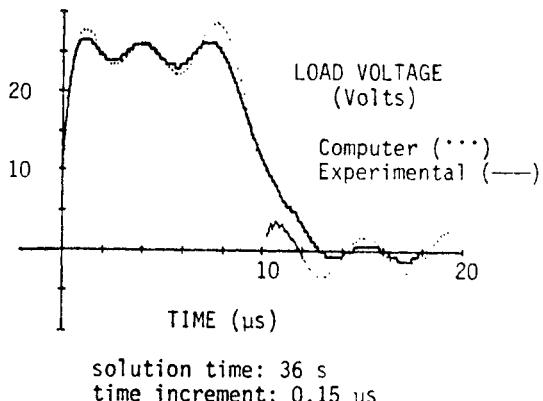


FIGURE 8. Computer & Experimental Voltage Across The Nonlinear Load

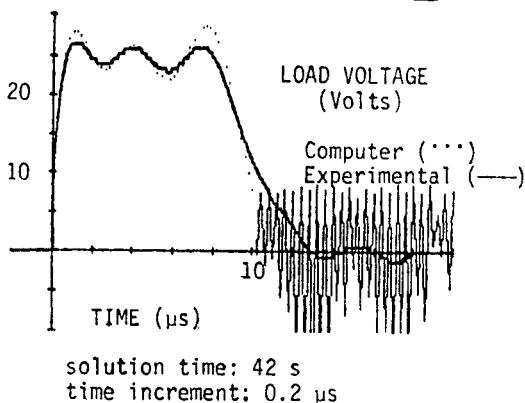


FIGURE 9. Computer & Experimental Voltage Across The Nonlinear Load

## CONCLUSION

A technique has been presented of microcomputer modeling which can be used in the design of linear and nonlinear circuits. This modeling technique provides a good approximation of the desired waveforms. Its greatest utility is in selecting component values for a particular pulser circuit and being able to see the results before the circuit is actually built. The advantage of this technique is that the circuit modeling can be run on a microcomputer found in most laboratories or universities, the solution times are reasonable and the program is simple.

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